$$= \frac{\left(\left(\underline{\sqrt{2+\sqrt{2}}} + \mathbf{i}\underline{\sqrt{2-\sqrt{2}}}\right)^2\right)}{}$$

$$\left(\cancel{2}+\underline{\sqrt{2}}-(\cancel{2}-\underline{\sqrt{2}})\right)+2i\sqrt{2+\sqrt{2}}\sqrt{2-\sqrt{2}}$$

$$2\sqrt{2} \quad +2i\underbrace{\sqrt{4-2}}_{\sqrt{2}}$$

$$\left[2\sqrt{2}(1+i)\right]^{4.} \qquad \left((1+i)^2\right)^2$$

$$= \frac{1}{64}\left(\frac{(\sqrt{3}-\mathbf{i})^2}{(1-\mathbf{i})^2}\right)^6$$

$$= \frac{1}{64}\left(\frac{2-2\sqrt{3}\mathbf{i}}{-2\mathbf{i}}\right)^6$$

$$= \frac{1}{64}(\sqrt{3}+\mathbf{i})^6$$

$$\frac{(2-2\sqrt{3}i)^6}{(-2i)^6}$$

$$(-2i)^6 = (-2)^6 i^6 = 2^6(-1)$$

$$= -2^6$$

$$= -64$$

$$i^6 = i^2 \cdot i^4 \quad i^4 = 1$$

$$= -1$$

$$(2-2\sqrt{3}i)^6 = 2^6\left[(1-\sqrt{3}i)^2\right]^3$$

$$(1-\sqrt{3}i)^2 = (1-\sqrt{3}i)(1-\sqrt{3}i)$$

$$= 1 - \sqrt{3}i - \sqrt{3}i + \underbrace{(\sqrt{3}i)^2}_{-3}$$

$$= 1 - 3 - 2\sqrt{3}i$$

$$= -2 - 2\sqrt{3}i$$

$$(-2-2\sqrt{3}i)^3 = (-2)^3 \underbrace{(1+\sqrt{3}i)^3}$$

1. $\dfrac{1+\mathbf{i}\sqrt{2}}{\sqrt{2}-\mathbf{i}}$ $= \dfrac{1+i\sqrt{2}}{\sqrt{2}-i} \cdot \dfrac{\sqrt{2}+i}{\sqrt{2}+i} = \dfrac{\cancel{\sqrt{2}}+i+i2-\cancel{\sqrt{2}}}{2+1}$

$= \dfrac{3i}{3} = i$

2. $\left(\dfrac{1-\mathbf{i}}{1+\mathbf{i}}\right)^2$ $= \left(\dfrac{(1-i)(1-i)}{\underbrace{(1+i)(1-i)}_{2}}\right)^2 = \dfrac{(-2i)^2}{4}$

$= -1$

3. $\mathbf{i}+\dfrac{1}{\mathbf{i}}$

$i+\dfrac{1}{i} = i+\dfrac{1}{i}\cdot\dfrac{-i}{-i}$

$= i+(-i) = 0$

$\overline{(i)} = -i$

$\overline{0+1.i} = 0-1.i$

$$\frac{1}{a+bi} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi}$$

$$(a+bi)(a-bi)$$

$$= \frac{a-bi}{a^2+b^2}$$

$$a^2 - \cancel{abi} + \cancel{bai} - b^2i^2 = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} \cdot i$$

$$a^2 - b^2 \underbrace{i^2}_{-1}$$

$$\mathrm{Im}\left(\frac{1}{a+bi}\right) = -\frac{b}{a^2+b^2}$$

$$a^2+b^2$$